Lecture 5
Plasma Acceleration

(Ch. 6, 4, 9 of UP-ALP)

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Motivations

• The “Livingston plot” shows the signs of saturation, highlighting the need for the next breakthrough in accelerator technology.

• Traditionally, accelerating structures have been made from metal (normal conductive or super-conductive) and are typically limited in their accelerating gradient to $E_z < 100$ MeV/m.
• The “accelerating structures” made from a material that is already “damaged” (e.g., plasma) will not exhibit the same limitations due to the material’s properties.
• Plasma acceleration was first proposed by T. Tajima and J. Dawson in 1979, which was, in fact, too early for laser and beam technologies to be ready to realize the proposed approach.
Laser Electron Accelerator

T. Tajima and J. M. Dawson

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(Received 9 March 1979)

An intense electromagnetic pulse can create a weak of plasma oscillations through the action of the nonlinear ponderomotive force. Electrons trapped in the wake can be accelerated to high energy. Existing glass lasers of power density $10^{18}$W/cm$^2$ shone on plasmas of densities $10^{18}$ cm$^{-3}$ can yield gigaelectronvolts of electron energy per centimeter of acceleration distance. This acceleration mechanism is demonstrated through computer simulation. Applications to accelerators and pulsers are examined.

Acceleration of Electrons by the Interaction of a Bunched Electron Beam with a Plasma

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(Received 20 December 1984)

A new scheme for accelerating electrons, employing a bunched relativistic electron beam in a cold plasma, is analyzed. We show that energy gradients can exceed 1 GeV/m and that the driven electrons can be accelerated from $\gamma_0 mc^2$ to $3\gamma_0 mc^2$ before the driving beam slows down enough to degrade the plasma wave. If the driving electrons are removed before they cause the collapse of the plasma wave, energies up to $4\gamma_0 mc^2$ are possible. A noncollinear injection scheme is suggested in order that the driving electrons can be removed.

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Plasma Oscillations

• Applying Gauss’s law along the integral contour:

\[ \oint_{\partial \Omega} \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0} \int_{\Omega} \rho dV \]

\[-E \times A(\text{top}) + 0(\text{side}) + 0 \times A(\text{bottom}) = \frac{1}{\varepsilon_0} (-enxA)\]

• An electric field produced by displaced charges is

\[ E = \frac{nex}{\varepsilon_0} \]

• Writing an equation for the electrons’ non-relativistic motion:

\[ F = ma \frac{d^2x}{dt^2} = -eE = -\frac{ne^2x}{\varepsilon_0} \]

• Expression for the oscillation frequency:

\[ \omega_p^2 = \frac{ne^2}{\varepsilon_0 m}, \quad f_p = \frac{\omega_p}{2\pi} \approx 9000 \times (n[\text{cm}^{-3}])^{1/2} \quad [\text{Hz}] \]

[Ex] For \( n \approx 10^{16} \text{ cm}^{-3} \), \( f_p \approx 900 \text{ GHz} \)
Maximum Field in Plasma

- Imagine that the plasma oscillation is excited by a charged object moving with velocity $c$.

$$x \sim \lambda_p \sim \frac{c}{\omega_p}$$

- This results in the following for the maximum field,

$$E_{\text{max}} \sim \frac{nec}{\epsilon_0 \omega_p} = \frac{mc\omega_p}{e}$$

- Or equivalently

$$eE_{\text{max}} \sim mc^2 \frac{\omega_p}{c}$$

- The maximum available accelerating gradient is

$$eE_{\text{max}} \approx 1 \frac{\text{eV}}{\text{cm}} \times (n [\text{cm}^{-3}])^{1/2}$$

[Ex] For $n \sim 10^{18} \text{ cm}^{-3}$, $eE_{\text{max}} \sim 1 \text{ GeV/cm}$
Early Steps of Plasma Acceleration

• The plasma wavelength is

\[ \lambda_p = \frac{c}{f_p}, \quad \text{or} \quad \lambda_p \approx 0.1 \text{ mm} \sqrt{\frac{10^{17}}{n[\text{cm}^{-3}]}} \]

[Ex] To achieve 1 GeV/cm, we need \( n \approx 10^{18} \text{ cm}^{-3} \), which corresponds to \( \lambda_p \approx 30 \mu\text{m} \) (or around 100 fs)

• Thus short sub-100-fs pulses are needed to excite plasma towards GeV/cm accelerating gradient.

• In the absence of such short pulses, in the late 1970s and early 1980s, other methods of plasma excitation were suggested such as the Plasma Beat Wave Accelerator (PBWA), and the Self-Modulated Laser WakeField Accelerator (SMLWFA)

  • PBWA: Two laser pulses with broad envelopes (a) and frequencies differing by \( \omega_p \) overlap to create a beating at the plasma’s frequency (b). This combined laser pulse is sent into plasma where it creates plasma excitation (c).

  • SMLWFA: Only a single laser pulse is sent into the plasma (a), where an instability results in a self-modulation of the long laser pulse at \( \lambda_p \) (b), which again creates plasma excitation at wavelength \( \lambda_p \) (c).
As a result of beam and laser technologies development, short sub-ps pulses of laser or beams became available and thus prompted rapid progress of plasma acceleration.

Plasma WakeField Acceleration (PWFA)

- Short, high-energy particle bunch
- \( \lambda_p \)

Laser WakeField Acceleration (LWFA)

- Short laser pulse of high intensity
- \( \lambda_p \)

[Note] There are far fewer PWFA than LWFA experiments being performed worldwide. This is because there are far fewer facilities that can provide the high-current, highly relativistic charged particle beams that are needed for such experiments. The two main facilities are at the SLAC National Accelerator Laboratory and the Brookhaven National Laboratory, both in the United States.
Basics of Lasers/Photon Beams
Laser Pulse Intensity

- Laser intensity (in a vacuum) is defined (in SI unit) as

\[ I = \frac{1}{2} \varepsilon_0 E_{max}^2 c = \text{Energy/Volume} \times \text{Length/Time} \]

- The intensity \( I \) is usually measured in Watts per cm\(^2\).

- The corresponding relation between fields and intensity is

\[ E_{max} \left[ \frac{V}{cm} \right] \approx 2.75 \times 10^9 \left( \frac{I}{10^{16} \text{W/cm}^2} \right)^{1/2} \]

\[ B_{max} \left[ \text{Gauss} \right] \approx 9.2 \times 10^6 \left( \frac{I}{10^{16} \text{W/cm}^2} \right)^{1/2} \]

\[ E(t) = B(t)c, \quad E_{max} = B_{max}c \]

for EM wave in vacuum.
Atomic Intensity

- In order to develop a quantitative understanding of laser intensity values, it is best to compare the field of an intense laser with atomic fields - particularly with the field in a hydrogen atom.
- Bohr radius is given by

$$a_B = \frac{\hbar^2}{me^2} = 5.3 \times 10^{-9} \text{ cm}$$

and corresponding electric field is

$$E_a = \frac{e^2}{4\pi\varepsilon_0a_B^2} \approx 5.1 \times 10^{11} \text{ V/m}$$

- The corresponding atomic intensity is thus equal to

$$I_a = \frac{\varepsilon_0cE_a^2}{2} \approx 3.51 \times 10^{16} \text{ W/cm}^2$$

- A laser with intensity higher than the above will ionize gas immediately. However, ionization can occur well below this threshold due to multiphoton effects or tunneling ionization.
Progress in Laser Peak Intensity

Around $2 \times 10^{29} \text{ W/cm}^2$: the Schwinger intensity limit, when the laser field can produce $e^+ e^-$ pairs from a vacuum.

Relativistic optics case: electrons become relativistic in the laser field

$$a_0 = \frac{eE_0}{m_e \omega c} = \frac{\text{Velocity gained by electron in one laser cycle}}{\text{Speed of light}}$$

$$I_a = \frac{\varepsilon_0 c E_a^2}{2} \approx 3.51 \times 10^{16} \frac{W}{\text{cm}^2}$$

CPA: chirped pulse amplification
Chirped Pulse Amplification

• Invented for lasers by G. Mourou and D. Strickland, *chirped pulse amplification* (or CPA) is a process that stretches the short pulse in time, amplifies the now-longer laser pulse and then compresses the amplified pulse.

• This multi-step process relieves the optical component of laser amplifiers from pulsed power, thus reducing nonlinear effects and avoiding material damage.

• The stretching and compressing of the pulses are based on the time(path length)–energy(frequency) correlation property of the laser pulse.

\[ \Delta f \sim \frac{1}{\tau} \]

Angle of reflection from the grating depends on the wavelength

A low peak power
Laser Pulse Stretcher/Compressor

- Compression of laser pulses relies on the dependence of the light reflection angle from the grating on the light wavelength. → Path length through the system depends on the light’s wavelength.

Pulse Stretcher
Longer wavelength → Shorter path length → Ahead of the pulse

Pulse Compressor
Longer wavelength → Longer path length → Ragging of the pulse
Achieving ultra-short electron bunches is usually done at higher energies, when the beam is relativistic and space charge effects are less severe.

A typical arrangement that can compress the bunch is a beamline made of four bending magnets of opposite polarity.

Particle velocities are nearly constant at c (Energy difference is different from velocity difference!)

In this chicane, the time of flight (or equivalently the path length) is different for different energies.
OPCPA: Optical Parametric CPA

- Its principle is based on nonlinear properties of crystals. When it is subjected to radiation of wavelength $\omega_s$, it generates radiation at two frequencies $\omega_1$ and $\omega_2$ where, as energy is conserved, $\omega_1 + \omega_2 = \omega_s$.

- In optical parametric amplification, the input consists of two beams: the *pump* at $\omega_s$ and the *signal* at $\omega_1$. The OPA output is the amplified $\omega_1$ beam and weakened $\omega_s$ beam, plus an additional *idler* (게으름뱅이) beam at $\omega_2$. 

![Diagram of OPCPA](image-url)
Types of ionization

- Potential well of an electron in an atom

With somewhat larger laser field intensity, Tunneling Ionization will turn into another ionization mechanism called Barrier Suppression Ionization (BSI).

\[
I_c = \frac{I_a}{256}
\]

\[
V(x) = -\frac{e^2}{3x} - eEx \quad \text{(Gaussian)}
\]

\[
x_{\max} = \left(\frac{e}{E}\right)^{1/2}
\]

\[
V_{\max} = 2\left(\frac{e^3E}{1}\right)^{1/2}
\]

13.6 eV
The spatial contrast: the ratio of intensity at the laser focus to the intensity outside of the focus.

For CPA-compressed pulses, it is appropriate to introduce the notion of the temporal contrast ratio a function of time given by the ratio of the peak laser intensity to the intensity in the front or back of the pulse.

The high contrast ratio is often the key parameter in plasma acceleration, as even a relatively low intensity can either ionize or destroy the target long before the arrival of the main high-intensity short pulse.

It is typical that a short sub-ps pulse (main) is accompanied by many tens of ps low-intensity pulses, as well as short pre-pulses or post-pulses, which are typically caused by nonlinear properties of the elements of the CPA system and non-ideal properties of the initial laser pulse.
Laser Plasma Acceleration
The concept of laser-plasma acceleration

- A powerful laser pulse enters gas, which can either be pre-ionized or not.
- The contrast ratio of the laser is not infinite, and so the ionization front starts in the gas at the front tail of the laser pulse, much in advance of the arrival of the main laser pulse.

The electrons of the plasma can be trapped in the wave and then accelerated (self-injection).

Maximum acceleration can occur when the laser pulse causes total separation of the electrons and ion charges of the plasma; this regime is nonlinear (or blow-out).

The cavity that is formed in the plasma and can trap and accelerate electrons is called a bubble.

Usually, electrons are trapped and accelerated in the first bubble.
The formation of a bubble is the result of ponderomotive force.

Let’s consider homogenous laser field,

\[ E = E_0 \cos(\omega t) \]

- The corresponding transverse motion is

\[ \frac{d^2 y}{dt^2} = \frac{F}{m} = \frac{qE}{m} = \frac{qE_0}{m} \cos(\omega t) \quad \rightarrow \quad y(t) = -\frac{qE_0}{m\omega^2} \cos(\omega t) \]

We then assume that the field has a gradient in transverse direction,

\[ E = E_0(y) \cos(\omega t) \approx E_0 \cos(\omega t) + y \frac{\partial E_0}{\partial y} \cos(\omega t) \approx E_0 \cos(\omega t) + \left( -\frac{qE_0}{m\omega^2} \cos(\omega t) \right) \frac{\partial E_0}{\partial y} \cos(\omega t) \]

\[ \langle F \rangle_t = \langle qE \rangle_t = \left\langle \left( -\frac{qE_0}{m\omega^2} \cos(\omega t) \right) \frac{\partial E_0}{\partial y} \cos(\omega t) \right\rangle = -\frac{1}{2} \frac{q^2}{m\omega^2} E_0 \frac{\partial E_0}{\partial y} = -\frac{q^2}{4m\omega^2} \frac{\partial E_0^2}{\partial y} \]
The ponderomotive force is proportional to the gradient of the laser intensity:

\[ I \propto E_0^2; \quad \langle F_t \rangle \propto -q^2 \frac{\partial I}{\partial y} \]

The direction of the force is such that the ponderomotive force pushes electrons out from the high intensity region.

We also note that the ponderomotive force is independent on the sign of the charge.
Ponderomotive force: For Alternating Focusing

• For sinusoidally varying focusing system:

\[ x'' + \kappa_0^2 \sin(k_p z) x = 0 \]

• The motion can be broken down into two components, one which contains the small amplitude fast oscillatory motion (the perturbed part of the motion), and the other that contains the slowly varying or secular, large amplitude variations in the trajectory.

\[ x = x_{osc} + x_{sec} \]

• We assumed

\[ x''_{osc} \gg x''_{sec}, \quad |x_{osc}| \ll |x_{sec}| \]

\[ x''_{osc} + \kappa_0^2 \sin(k_p z) x_{sec} = 0 \]

• Inhomogeneous component of the above solution is

\[ x_{osc} = \sin(k_p z) \frac{\kappa_0^2}{k_p^2} x_{sec} \]

\[ \frac{|x_{osc}|}{|x_{sec}|} \ll 1, \quad \frac{\kappa_0^2}{k_p^2} \ll 1 \]
Ponderomotive force: For Alternating Focusing

- The original equation of motion is modified as

\[ x'' = -\kappa_0^2 \sin(k_p z) x \simeq -\kappa_0^2 \sin(k_p z) (x_{sec} + x_{osc}) \simeq -\kappa_0^2 \sin(k_p z) \left[ 1 + \sin(k_p z) \frac{\kappa_0^2}{k_p^2} \right] x_{sec} \]

- By averaging over one period of the fast oscillation,

\[ \langle x'' \rangle = x''_{sec} \simeq -\frac{1}{2} \kappa_0^2 \frac{\kappa_0^2}{k_p^2} x_{sec} \]

- In standard simple harmonic oscillator form:

\[ x''_{sec} + \frac{\kappa_0^4}{2k_p^2} x_{sec} = 0 \rightarrow x_{sec} = A \cos \left( \frac{\kappa_0^2}{\sqrt{2}k_p} z \right) + B \sin \left( \frac{\kappa_0^2}{\sqrt{2}k_p} z \right) \]

- The full solution with constants of integration is

\[ x = \left[ 1 + \sin(k_p z) \frac{\kappa_0^2}{k_p^2} \right] \left[ A \cos \left( \frac{\kappa_0^2}{\sqrt{2}k_p} z \right) + B \sin \left( \frac{\kappa_0^2}{\sqrt{2}k_p} z \right) \right] \]
Ponderomotive force: For Alternating Focusing

Phase advance per period:

\[ \mu \simeq k_{sec} L_p = \frac{k_0^2 L_p}{\sqrt{2k_p}} = \sqrt{2\pi} \frac{k_0^2}{k_p^2} \]

\[ k_p = \frac{2\pi}{L_p} \]
Nonlinear Regime

- Ponderomotive force plays a key role in the formation of the accelerating bubble in the nonlinear (also called blow-out) regime.

- The ponderomotive force of a short (typically ~50 fs) and intense (typically ~10^{18} W/cm^2) laser pulse expels plasma electrons while heavier ions stay at rest.

- The expelled electrons are immediately attracted back to the ions, forming the first bubble.

- The gradient of the density of electrons creates spatial oscillation of the electric field within plasma (reaching ~100 GV/m) which can accelerate particles.
Nonlinear Regime

• Having formed the first bubble, the electrons continue their oscillations around the ions, but their motions quickly become incoherent and so the second and subsequent bubbles gradually become smaller.

• Only the first bubble (and sometimes the second) is useful for acceleration.
Wave breaking: Self-injection

- The high accelerating gradient in the plasma is useful only if a particle beam can be injected into the bubble.
- Luckily, self-injection of background plasma electrons into the plasma bubble can occur through the wave breaking phenomenon.
- Wave breaking transpires when, within the nonlinear regime, certain particles outrun the wave.

\[ y'' + k^2 y = 0 \]

Other methods:
- Injection of an external electron beam (challenging if the bunches are short)
- Using multiple laser pulses
- Mixing of gases with different ionization potentials
Wave breaking: Self-injection

- Simulation using PIC code (Prof. Hae-June Lee’s group/PNU)

In the moving window

Wave breaking and self-injection

Oscillation of accelerating electrons

Photon energy: 1 ~ 100 keV (Hard X-ray)

Betatron radiation produced by oscillating beams
Transverse fields in the bubble

- We assume the ions are heavy and stationary within the bubble. The ions produce a focusing force.

$$\int \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0} \int \rho dV$$

- Assuming cylindrical symmetry

$$F = -eE = -\frac{1}{2\varepsilon_0}ne^2r$$

- A relativistic electron will oscillate in the electric field as

$$\gamma m \frac{d^2r}{dt^2} = -\frac{1}{2\varepsilon_0}ne^2r \quad \Rightarrow \quad \frac{d^2r}{ds^2} = -\frac{1}{2\varepsilon_0\gamma mc^2}ne^2r = -\frac{\omega_p^2}{2\gamma c^2}r$$

$$r'' + \frac{\omega_p^2}{2\gamma c^2}r = 0$$

$$= k^2 = \omega^2/c^2$$

- The period of oscillation is thus given by

$$\omega = \frac{\omega_p}{\sqrt{2\gamma}}, \quad \lambda = \sqrt{2\gamma} \lambda_p$$

Only for electrons inside the bubble, which are ultra-relativistic.
Estimation of betatron radiation parameters

- The synchrotron radiation is the result of the charged particle leaving part of its fields behind (as the field cannot catch up with the motion of the particles)

- The part of field moving further away \((r)\) would be left behind. For \(\gamma \gg 1, \beta \approx 1,\)

\[
r = R \left( \frac{c}{v} - 1 \right) = R \frac{1 - \beta}{\beta} = R \frac{(1 - \beta)(1 + \beta)}{\beta(1 + \beta)} \approx R \frac{1 - \beta^2}{2} = \frac{R}{2\gamma^2}
\]
Estimation of betatron radiation parameters

- Based on the relativistic kinematics, we note that the radiation of relativistic particles is emitted into a cone with angular spread of $1/\gamma$.

Time for first photon to arrive at point B:

\[ t_1 = \frac{2R \sin(1/\gamma)}{c} \]

Time for the particle to arrive at point B:

\[ t_2 = \frac{2R}{\gamma v} \]

\[ \delta t = t_2 - t_1 = \frac{2R}{v} \left[ \frac{1}{\gamma} - \beta \sin(1/\gamma) \right] \approx \frac{2R}{v \gamma} (1 - \beta) \approx \frac{R}{c \gamma^3} \]

- Estimate the characteristic frequency of emitted photons as the inverse of the time duration of the light pulse.

\[ \omega_c \approx \frac{1}{\delta t} \approx \frac{c \gamma^3}{R} \rightarrow \frac{3}{2} \frac{c \gamma^3}{R} \]
Estimation of betatron radiation parameters

- Energy loss per unit length due to synchrotron radiation:

\[
\frac{dW}{ds} = \frac{2}{3} \frac{e^2 \gamma^4}{R^2}
\]

\[
\int dV \sim \int r^2 \sin \theta dr d\theta d\phi \sim \int r dr ds \rightarrow W \propto \frac{e^2}{r^2} ds
\]

- Number of photons emitted per unit length:

\[
\frac{dN}{ds} = \frac{1}{\epsilon_c} \frac{dW}{ds} = \frac{1}{\hbar \omega_c} \frac{dW}{ds} = \frac{\alpha \gamma}{R}
\]

Fine structure constant: \( \alpha = \frac{e^2}{(4\pi \epsilon_0) \hbar c} \approx 1/137 \)

- The radius of the curvature of the beam trajectory:

\( r_b \) = amplitude of beam oscillation in the bubble
\( \lambda \) = period of beam oscillation in the bubble

\[
R - R \cos(\theta/2) = r_b \\
R \sin(\theta/2) = \lambda/4 \\
R \approx \frac{\lambda^2}{32r_b} \rightarrow \frac{\lambda^2}{4\pi^2r_b}
\]
Estimation of betatron radiation parameters

- The radius of the curvature is
  \[ R = \frac{\lambda^2}{4\pi^2r_b} = \frac{\gamma\lambda_p^2}{2\pi^2r_b} \]

- Radiation wavelength for the laser plasma betatron source:
  \[ \lambda_c = \frac{2\pi c}{\omega_c} = \frac{2\pi c}{\frac{3}{2}c\frac{\gamma^3}{R}} = \frac{1}{3\pi} \frac{\lambda_p^2}{r_b} \frac{1}{\gamma} \]

- The number of photons emitted per \( \lambda \):
  \[ N_{\gamma} \approx \frac{dN}{ds} \lambda \approx \sqrt{2\gamma}2\pi^2\alpha \frac{r_b}{\lambda_p} \]

[Example] A beam is accelerated in the bubble with \( \lambda_p = 0.03 \) mm up to 1 GeV (\( \gamma = 2 \times 10^3 \)). In a very rough approximation, \( r_b \approx 1\% - 10\% \) of the bubble size \( \lambda_p \). Therefore, we may assume \( r_b = 0.001 \) mm.

\[ \lambda_c = 0.025 \text{ nm (} \sim 50 \text{ keV)} \rightarrow \text{Hard X-ray} \]
\[ N_{\gamma} \sim 0.3 \text{ per each accelerated electrons} \]

Considering that the accelerating bunch can carry tens of pC to nC charge, we can conclude that such a light source can generate many hard X-ray photons.
Compact radiation sources

- Laser-driven plasma accelerators can already generate electron beams with several GeV of energy, 10 fs bunch duration and 10-100 pC of charge per bunch. → potentially suitable for creating compact radiation sources.

- Challenges:
  - Repetition rate (~ Hz yet)
  - Wall-plug efficiency
  - Beam quality (energy spread and emittance)

- Outlook: All types of light sources (conventional + laser-driven) will continue to co-exist and national-scale facilities will be complemented by a variety of compact plasma acceleration based light sources.
Competing Effects

- As a laser pulse travels through the gas or plasma, several competing effects are taking place.

- **Diffraction**: A laser beam focused to a size of several tens um will diffract very fast.
- **Dephasing**: The gradual separation of the accelerating beam (which quickly become relativistic) from the laser (which propagates in gas or plasma slower than the speed of light).
- **Depletion**: The gradual decrease of the laser intensity.
- **Longitudinal compression** of the laser pulse by plasma wave
- **Self-focusing**: Due to the relativistic effect (the electrons of plasma at the axis become relativistic and have higher masses, affecting the plasma refraction coefficient)
- **Ionization-caused diffraction**: Gas on the axis where intensity is higher will be ionized first, affecting diffraction.

\[
\frac{\Theta}{2} w_0 = \frac{\lambda}{\pi} \\
\frac{\Theta}{2} \sim \frac{\omega_0}{z_R}
\]

Valid for \( w_0 \geq 2\lambda/\pi \)
So the dispersion relation is given by

\[ \omega^2 = \omega_{pe}^2 + k^2 c^2 \]

\[ \omega_{pe} = \frac{n_e e^2}{\epsilon_0 m_e} \]

**NOTES**

For transverse waves,

- Phase velocity:
  \[ v_p^2 = \frac{\omega^2}{k^2} = c^2 + \frac{\omega_{pe}^2}{k^2} > c^2 \]

- Group velocity:
  \[ v_g = \frac{d\omega}{dk} = \frac{c^2}{v_p} < c \]

- The index of refraction: \[ n^2 = \left( \frac{\omega}{\omega_p} \right)^2 = \left( \frac{kx}{\omega} \right)^2 = 1 - \frac{\omega_{pe}^2}{\omega^2} < 1. \]

- Since \( k^2 = \frac{\omega^2 - \omega_{pe}^2}{c^2} \)
  1. \( \omega > \omega_{pe} \): \( k \) is real so that the wave is propagating.
  2. \( \omega = \omega_{pe} \): \( k = 0 \) (cutoff)
  3. \( \omega < \omega_{pe} \): \( k \) is imaginary so that the wave is evanescent.

\[ e^{ikx} = e^{-|k|x} = e^{-\omega |x|} \]

where \( \delta = \frac{1}{|k|} = \frac{c}{\sqrt{\omega_{pe}^2 - \omega^2}} \): skin depth.
Laser guidance

- Capillary channel: A special density profile $n(r)$ will be formed to assist guiding the laser pulse for a significant distance (refraction-assisted).

- First realized at the Oxford University in 2006.
- Broke the GeV barrier in laser plasma acceleration.

Electrode

- Sapphire is far more scratch resistant than glass
- Sapphire is one of the hardest, most wear-resistant materials available after diamond
- Sapphire has much a higher IR and UV transmission than glass
- Sapphire is also more temperature & chemical resistant than glass
- Sapphire is also a good thermal conductor, much better than standard glasses
- Sapphire is a superior dielectric material with a high dielectric constant and low loss tangent

Capillary channel: A special density profile $n(r)$ will be formed to assist guiding the laser pulse for a significant distance (refraction-assisted).

Broke the GeV barrier in laser plasma acceleration.
Capillary plasma source

Power supply charges the capacitor up to 25 kV and the thyatron switch is triggered for conduction.

Rise time ~ 10 ns
Peak current ~ 275 A

50 ~ 300 Torr

[GIST, 2009]
Dephasing

• The laser pulse propagating through a medium (plasma) has \( v < c \) and the accelerating electrons that quickly become relativistic will soon dephase from the plasma wave.

• The group velocity of a laser pulse:

\[
v_g = c \sqrt{1 - \omega_p^2/\omega^2}
\]

• The dephasing occurs when an electron outruns the wave by a half of a period:

\[
(c - v_g)t_d = \frac{\lambda_p}{2}
\]

• The dephasing length:

\[
L_d = ct_d = \frac{\lambda_p}{2} \frac{1}{\left(1 - \sqrt{1 - \omega_p^2/\omega^2}\right)} 
\approx \frac{\lambda_p \omega^2}{\omega_p^2}
\]

[Example] For a laser with a wavelength of 1 \( \mu \text{m} \) and \( \lambda_p = 30 \text{ m} \), the dephasing length is \( L_d = 30 \text{ mm} \)

\[\rightarrow\] Needs multi-stage laser plasma acceleration!

\[\rightarrow\] How to preserve beam qualities during the multi-stage acceleration?
Beam-driven plasma acceleration

- Plasma can be excited not with a laser pulse, but with a short intense bunch of charged particles. The bubble will be formed due to the bunch’s field.
  - The driver beam has $v = c$, and thus dephasing of the witness beam from the driver is no longer an issue.
  - The driver beam can carry much more energy than a laser pulse.

- Disadvantage:

  $$R = \frac{E_{\text{witness}}}{E_{\text{drive}}} \leq 2 - \frac{N_{\text{witness}}}{N_{\text{drive}}}$$

Transformer ratio $R$ is at most two for longitudinally symmetric drive bunches. This upper limit can in principle be overcome by non-symmetric bunches, but this could be difficult.
Beam-driven plasma acceleration

**FFTB (SLAC)**

**FACET (SLAC)**

Transformer ratio limit → Proton-driven → AWAKE (CERN)
Laser-driven acceleration “without plasma”

Micro-fabricated dielectric laser (IR) accelerators


Affiliations | Contributions | Corresponding author

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Micro-fabricated dielectric laser (IR) accelerators ($Q_0 \to 0$)

http://www.youtube.com/watch?v=V89qvy8whxY

IR laser

THz laser

Physics World - the member magazine of the Institute of Physics

Tiny terahertz accelerator could rival huge free-electron lasers

Oct 12, 2015 @2 comments

Physicists in the US, Germany and Canada have built a miniature particle accelerator that uses terahertz radiation instead of radio waves to create pulses of high-energy electrons. A single accelerator module of the prototype is just 1.5 cm long and 1 mm thick, and the technology has the potential to create facilities that are much smaller than current radio-frequency (RF) accelerators. Potential applications include free-electron lasers, whereby the electrons are used to create coherent pulses of X-rays. However, the team cautions that much more work is needed to develop the technology so it can be used in medicine, particle physics and material science.